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Punctuated-equilibrium model of biological evolution is also a self-organized-criticality model of earthquakes

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Bak and Sneppen proposed a self-organized-criticality model to explain the punctuated equilibrium of biological evolution. The model, as it is, is a good self-organized-criticality model of earthquakes. Real earthquakes satisfy the required conditions of criticality; that is, power laws in (1) the size distribution of earthquakes, and (2) both the spatial and the temporal correlation functions.

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Bak and others [1] introduced a self-organized-criticality (SOC) model to explain the ubiquitous occurrence of fractal structures and $1/f$ noises in nature. Since their sandpile model was found to be a cellular-automaton version of the spring-block model of earthquakes by Burridge and Knopoff [2], seismology has become an important field of application of self-organized criticality [3–6].

Bak and Sneppen [7] recently proposed a SOC model to mimic punctuated equilibrium of biological evolution. In this paper, I point out that the Bak and Sneppen (BS) model can be applied to the dynamics of earthquake fault motion and that it is a mathematical presentation of the "barrier model" known in seismology. The BS model is a one-dimensional array of N sites. Each site represents a species, and is assigned initially a random barrier, B_i, between 0 and 1. The barrier is a measure of the fitness or survivability of the species. At each time step, the site with the lowest barrier is identified and the species at that site is mutated or assigned with a new random number. All species interact with each other through a food chain. The interaction is introduced by assigning new random numbers to the two nearest neighbor sites of the mutated site. This procedure is repeated.

Irrespective of the initial distribution, the system reaches a stationary state, displaying the self-organized-critical behavior. The model can easily be explained in seismological terms, too. The fitness landscape of the BS model is equivalent to the heterogeneous barrier distribution over a fault plane generating earthquakes. Mutation corresponds to rupture. In seismology, nonuniform distribution of strength over a fault plane is called "barriers" or "asperities," and is considered to cause the complex ruptures process of earthquakes [8,9]. Irregularity is either geometrical or in the physical and mechanical properties. We can consider the barrier in the BS model as the magnitude of strength relative to tectonic stress applied on a fault in the following earthquake fault model.

In the fault model, a rupture starts from the weakest site with the minimum barrier strength. When the site breaks, the stress in the neighborhood changes. This is modeled by assigning new random numbers to the three sites: the ruptured site and the two nearest neighbor sites. Rupture propagates as long as the new barriers are weaker than the threshold value of rupture. An earthquake, that is an avalanche of rupture, is defined to stop when the minimum barrier becomes stronger than the threshold. Another earthquake will start from the site with the minimum barrier after some time when the tectonic stress is increased again. This procedure is essentially the same as that of the biological evolution model.

The size distribution of earthquakes is known as the Gutenberg-Richter relation:

$$\log_{10}N(>M)=a-bM ,$$

where M is the magnitude and $N(>M)$ is the number of earthquakes with the magnitude greater than M. The b value is close to 1 [10]. The size of the ruptured zone S is related to the magnitude as [11]

$$\log_{10}S=M-3.7 ,$$

and thus,

$$N(>S)\sim S^{-b} ,$$

demonstrating that the size distribution of earthquakes follows a power law.

de Boer and others [12] argued that a power-law distribution of avalanches is not sufficient enough to claim that a model or a phenomenon is at criticality. They considered that power-law correlations in both space and time are at least required.

We have calculated these two-point spatial correlations and temporal correlations for Californian earthquakes. According to de Boer and others, the spatial correlation function $P_x(x)$ is defined as the probability that hypocenters of two successive earthquakes are separated by distance x. Two kinds of temporal correlation are considered. The first-return probability $P_1(t)$ is defined as the probability that, if an earthquake occurs at a given small region at time t_0, another earthquake occurs again at the same region for the first time at time t_0+t. The all-return probability $P_t(t)$ is the probability that earthquakes will occur at time t_0+t at the given region.

The ISC (International Seismological Center) data of Californian earthquakes from 1971 to 1985, for which a multifractal analysis has been done [13], are used. The area is chosen between 20° and 45° N latitude and between 100° and 125° W longitude, and 8992 earthquakes are included. The size of a small region is chosen to be 1°

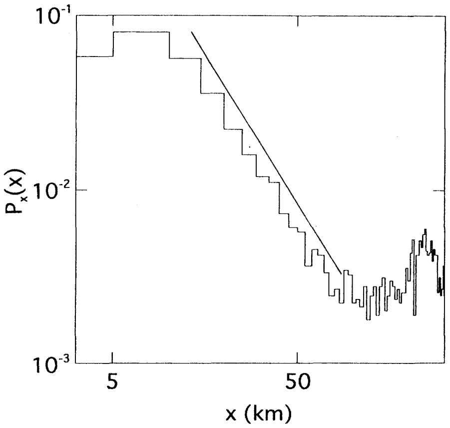

FIG. 1. The spatial correlation function $P_x(x)$ of hypocenters for Californian earthquakes between 1971 and 1985. The solid line has a slope of -1.7.

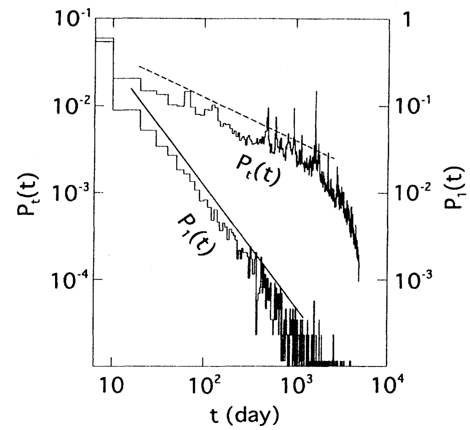

FIG. 2. The temporal correlation functions [the first-return $P_1(t)$ and the all-return $P_t(t)$] of Californian earthquakes (1971–1985). The continuous line has a slope of -1.4, and the dashed one that of -0.5.

latitude $\times 1°$ longitude. In the multifractal analysis, the scaling range of self-similarity was found to be limited between 6 and 60 km.

Results are shown in Figs. 1 and 2. In all cases, we observe power-law behavior:

$$P_x(x) \sim x^{-1.7} ,$$

$$P_1(t) \sim t^{-1.4} ,$$

$$P_t(t) \sim t^{-0.5} .$$

The exponent for $P_t(t)$, which is less than 1, also satisfies the requirements of criticality. It should be noted that the catalog includes aftershocks accompanying large earthquakes, while the present discrete-time model does not separate aftershocks from the main shock. The absence of aftershocks is a common weak point among SOC models of earthquakes. Ito and Matsuzaki [5], taking the relaxation process into account, presented a revised model which generates aftershocks. Power-law behaviors did not change significantly, excepting for the spatial correlation.

We pointed out that the simple SOC model of biological evolution by Bak and Sneppen can be considered as another SOC model of earthquakes, as well as the sandpile model by Bak and others. We showed that real earthquakes satisfy the requirements of criticality claimed by de Boer and others, that is, power laws in (i) the size distribution of avalanches, (ii) the spatial correlation function, and (iii) the temporal correlation functions.

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